

# Chaplygin gas with non-adiabatic perturbations

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May 2, 2006

## Abstract

Perturbations in a Chaplygin gas, characterized by an equation of state  $p = -A/\rho$ , may acquire non-adiabatic contributions if spatial variations of the parameter  $A$  are admitted. This feature is shown to be related to a specific internal structure of the Chaplygin gas. We investigate how perturbations of this type modify the adiabatic sound speed. A reduction of the effective sound speed compared with the adiabatic value is expected to suppress oscillations in the matter power spectrum. This text is an abridged version of the following reference: W. Zimdahl and J.C. Fabris, *Classical and Quantum Gravity*, **22**, 4311(2005).

## 1 Introduction

Since 1998 [1] a growing number of observational data has backed up the conclusion that the expansion rate of our present Universe is increasing. According to our current understanding on the basis of Einstein's General Relativity, such a dynamics requires a cosmic substratum with an effective negative pressure. To clarify the physical nature of this substratum is one of the major challenges in cosmology. Most of the approaches in the field rely on a two-component picture of the cosmic medium with the (at present) dynamically dominating "Dark Energy" (DE), equipped with a negative pressure, which contributes roughly

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70% to the total energy density, and with pressureless (Cold) "Dark Matter" (CDM), which contributes roughly 30% (see, e.g. [2] and references therein). Usually, these components are assumed to evolve independently, but more general interacting models have been considered as well [3].

A single-component model which has attracted some interest as an alternative description is a Chaplygin gas [4]. The Chaplygin gas which is theoretically based in higher dimensional theories [5], has been considered as a candidate for a unified description of dark energy and dark matter [6, 7, 8, 9, 10, 11]. Its energy density smoothly changes from that of matter at early times to an almost constant value at late times. It interpolates between a phase of decelerated expansion, necessary for structure formation to occur, and a subsequent period in which the dynamically dominating substratum acts similarly as a cosmological constant, giving rise to accelerated expansion.

The Chaplygin gas combines a negative pressure with a positive sound velocity. This sound speed is negligible at early times and approaches the speed of light in the late-time limit. A sound speed of the order of the speed of light has implications which apparently disfavor the Chaplygin gas as a useful model of the cosmic medium. In particular, it should be connected with oscillations of the medium on small (sub-horizon) scales. The fact that the latter are not observed has led the authors of [12] to the conclusion that Chaplygin gas models of the cosmic medium are ruled out as competitive candidates.

However, these conclusions rely on the assumption of an adiabatic cosmic medium. It has been argued that there might exist entropy perturbations, so far not taken into account, which may change the result of the adiabatic perturbation analysis [13, 14]. A problem here is the origin of non-adiabatic perturbations which should reflect the internal structure of the cosmic medium. The latter is unknown but it may well be more complicated than suggested by the usually applied simple (adiabatic) equations of state. Generally, non-adiabatic perturbations will modify the adiabatic sound speed.

The purpose of this paper is to study a simple model of non-adiabatic perturbations in a Chaplygin gas. Starting with a two-component description of the cosmic medium we demonstrate that any Chaplygin gas in a homogeneous and isotropic Universe can be regarded as being composed of another Chaplygin gas which is in interaction with a pressureless fluid such that the background ratio of the energy densities of both components is constant. Spatial perturbations of this ratio then give rise to non-adiabatic pressure perturbations which may be related to fluctuations of the equation of state parameter  $A$ . The influence of these perturbations on the sound speed is investigated.

## 2 Interacting two-fluid dynamics

Much of the appeal of a Chaplygin gas model of the cosmic substratum is due to the circumstance that it represents a unified description of dark energy

and dark matter within a one component model. In order to describe non-adiabatic features which are the manifestation of an internal structure of the medium it will be instructive, however, to start the discussion by establishing basic relations of a two-fluid description. The point here is that we shall reveal an equivalent two-component picture of any one-component Chaplygin gas (for different two-component decompositions see [11, 15]). This circumstance can be used to introduce a simple model for an internal structure which gives rise to non-adiabatic pressure perturbations.

## 2.1 General setting

We assume the cosmic medium to behave as a perfect fluid with an energy-momentum tensor

$$T^{ik} = \rho u^i u^k + p h^{ik}, \quad h^{ik} = g^{ik} + u^i u^k. \quad (1)$$

Our approach is based on the decomposition

$$T^{ik} = T_1^{ik} + T_2^{ik} \quad (2)$$

of the total energy-momentum into two parts with ( $A = 1, 2$ )

$$T_A^{ik} = \rho_A u_A^i u_A^k + p_A^* h_A^{ik}, \quad h_A^{ik} = g^{ik} + u_A^i u_A^k, \quad T_{A;k}^{ik} = 0. \quad (3)$$

For our purpose it is convenient to split the (effective) pressures  $p_1^*$  and  $p_2^*$  according to

$$p_1^* \equiv p_1 + \Pi_1, \quad p_2^* \equiv p_2 + \Pi_2. \quad (4)$$

This procedure allows us to introduce an arbitrary coupling between both fluids by the requirement

$$\Pi_1 = -\Pi_2 \equiv \Pi \quad \Leftrightarrow \quad p_1^* = p_1 + \Pi, \quad p_2^* = p_2 - \Pi. \quad (5)$$

Under this condition the apparently separate energy-momentum conservation  $T_{A;k}^{ik} = 0$  in (3) is only formal. Then, for a homogeneous and isotropic universe with  $u_1^i = u_2^i = u^i$  and  $u_{;a}^a = 3H$ , where  $H$  is the Hubble rate, the individual energy balance equations are coupled and take the form

$$\dot{\rho}_1 + 3H(\rho_1 + p_1) = -3H\Pi \quad \Leftrightarrow \quad \dot{\rho}_1 + 3H(\rho_1 + p_1^*) = 0, \quad (6)$$

and

$$\dot{\rho}_2 + 3H(\rho_2 + p_2) = 3H\Pi \quad \Leftrightarrow \quad \dot{\rho}_2 + 3H(\rho_2 + p_2^*) = 0, \quad (7)$$

where  $\rho = \rho_1 + \rho_2$  and  $p = p_1 + p_2$ . The total adiabatic sound velocity may be split according to

$$\frac{\dot{p}}{\dot{\rho}} = \frac{\dot{\rho}_1}{\dot{\rho}} \frac{\dot{p}_1}{\dot{\rho}_1} + \frac{\dot{\rho}_2}{\dot{\rho}} \frac{\dot{p}_2}{\dot{\rho}_2} = \frac{\dot{\rho}_1}{\dot{\rho}} \frac{\dot{p}_1^*}{\dot{\rho}_1} + \frac{\dot{\rho}_2}{\dot{\rho}} \frac{\dot{p}_2^*}{\dot{\rho}_2}. \quad (8)$$

We emphasize that so far the equations of state and the nature of the interaction between the components are left unspecified.

## 2.2 Matter perturbations

To study perturbations about the homogeneous and isotropic background we introduce the following quantities. We define the total fractional energy density perturbation

$$D \equiv \frac{\hat{\rho}}{\rho + p} = -3H \frac{\hat{\rho}}{\dot{\rho}} , \quad (9)$$

and the energy density perturbations for the components ( $A = 1, 2$ )

$$D_A \equiv \frac{\hat{\rho}_A}{\rho_A + p_A^*} = -3H \frac{\hat{\rho}_A}{\dot{\rho}_A} . \quad (10)$$

Likewise, the corresponding pressure perturbations are

$$P \equiv \frac{\hat{p}}{\rho + p} = -3H \frac{\hat{p}}{\dot{\rho}} = -3H \frac{\dot{p}}{\dot{\rho}} \frac{\hat{p}}{\dot{p}} , \quad (11)$$

and

$$P_A \equiv \frac{\hat{p}_A}{\rho_A + p_A^*} = -3H \frac{\hat{p}_A}{\dot{\rho}_A} = -3H \frac{\dot{p}_A}{\dot{\rho}_A} \frac{\hat{p}_A}{\dot{p}_A} . \quad (12)$$

One also realizes that

$$D = \frac{\dot{\rho}_1}{\dot{\rho}} D_1 + \frac{\dot{\rho}_2}{\dot{\rho}} D_2 \quad (13)$$

and

$$P = \frac{\dot{\rho}_1}{\dot{\rho}} P_1 + \frac{\dot{\rho}_2}{\dot{\rho}} P_2 . \quad (14)$$

In terms of these quantities the non-adiabatic pressure perturbations

$$P - \frac{\dot{p}}{\dot{\rho}} D = -3H \frac{\dot{p}}{\dot{\rho}} \left[ \frac{\hat{p}}{\dot{p}} - \frac{\hat{\rho}}{\dot{\rho}} \right] \quad (15)$$

are then generally characterized by

$$\begin{aligned} P - \frac{\dot{p}}{\dot{\rho}} D &= \frac{\dot{\rho}_1}{\dot{\rho}} \left( P_1 - \frac{\dot{p}_1}{\dot{\rho}_1} D_1 \right) + \frac{\dot{\rho}_2}{\dot{\rho}} \left( P_2 - \frac{\dot{p}_2}{\dot{\rho}_2} D_2 \right) \\ &\quad + \frac{\dot{\rho}_1 \dot{\rho}_2}{(\dot{\rho})^2} \left[ \frac{\dot{p}_2}{\dot{\rho}_2} - \frac{\dot{p}_1}{\dot{\rho}_1} \right] [D_2 - D_1] . \end{aligned} \quad (16)$$

The first two terms on the right-hand side describe internal non-adiabatic perturbations within the individual components. The last term takes into account non-adiabatic perturbations due to the two-component nature of the medium.

### 3 The Chaplygin gas

#### 3.1 Two-component interpretation

The Chaplygin gas is characterized by an equation of state

$$p = -\frac{A}{\rho} , \quad (17)$$

where  $p$  is the pressure and  $\rho$  is the energy density of the gas.  $A$  is a positive constant. This equation of state gives rise to the energy density [6]

$$\rho = \sqrt{A + \frac{B}{a^6}} , \quad (18)$$

where  $B$  is another (positive) constant. Now, let us rename the constants according to

$$A = A_2 (1 + \kappa) , \quad B = B_2 (1 + \kappa)^2 . \quad (19)$$

For any constant, non-negative  $\kappa$  the quantities  $A_2$  and  $B_2$  are constant and non-negative as well. So far, no physical meaning is associated with these constants. It is obvious, that this split allows us to write the energy density (18) as

$$\rho = (1 + \kappa) \sqrt{\frac{A_2}{1 + \kappa} + \frac{B_2}{a^6}} . \quad (20)$$

This means,  $\rho$  can be regarded as consisting of two components,  $\rho_1$  and  $\rho_2$ ,

$$\rho = \rho_1 + \rho_2 , \quad (21)$$

with

$$\rho_1 = \sqrt{\frac{A_2 \kappa^2}{1 + \kappa} + \frac{B_2 \kappa^2}{a^6}} \quad (22)$$

and

$$\rho_2 = \sqrt{\frac{A_2}{1 + \kappa} + \frac{B_2}{a^6}} , \quad (23)$$

where  $\rho_1 = \kappa \rho_2$  is valid, i.e., the ratio  $\rho_1/\rho_2 = \kappa$  is constant. The corresponding (effective) equations of state are (the notations are chosen in agreement with the formalism of subsection 2.1)

$$p_1^* = -\frac{A_2 \kappa^2}{(1 + \kappa) \rho_1} = -\frac{A_2 \kappa}{(1 + \kappa) \rho_2} , \quad (24)$$

and

$$p_2^* = -\frac{A_2}{(1 + \kappa) \rho_2} . \quad (25)$$

This implies  $p_1^* = \kappa p_2^*$ , i.e., the effective pressures of both components differ by the same constant which also characterizes the ratio of both energy densities. It is further convenient to introduce a quantity

$$p_2 \equiv -\frac{A_2}{\rho_2} . \quad (26)$$

This quantity differs from  $p_2^*$  by

$$p_2^* - p_2 = \frac{\kappa A_2}{(1 + \kappa) \rho_2} = -p_1^* . \quad (27)$$

Defining also a quantity  $\Pi$  (the use of the same symbol as in (5) will be justified in the next step) by

$$\Pi \equiv p_1^* = -\frac{\kappa}{1 + \kappa} \frac{A_2}{\rho_2} = -\frac{\kappa}{1 + \kappa} \frac{A}{\rho} , \quad (28)$$

one checks by direct calculation that the following relations are valid:

$$\dot{\rho}_1 + 3H\rho_1 = -3H\Pi \quad \Leftrightarrow \quad \dot{\rho}_1 + 3H(\rho_1 + p_1^*) = 0 , \quad (29)$$

and

$$\dot{\rho}_2 + 3H(\rho_2 + p_2) = 3H\Pi \quad \Leftrightarrow \quad \dot{\rho}_2 + 3H(\rho_2 + p_2^*) = 0 . \quad (30)$$

At this point it becomes clear what we have obtained by the formal manipulations in Eq.(19). It describes a two-component system in which the components interact with each other. The role of the interaction is to keep the ratio  $\kappa$  of the energy densities of both components constant. In other words, *any given Chaplygin gas can be thought as being composed of another Chaplygin gas which is in interaction with a pressureless fluid such that the energy density ratio of both components is fixed.*

The components have sound velocities which are different from each other and different from the overall sound velocity. For the latter we have

$$\frac{\dot{p}}{\dot{\rho}} = -\frac{p}{\rho} = \frac{A}{\rho^2} = \frac{A}{A + \frac{B}{a^6}} . \quad (31)$$

The sound velocities of the components are  $\dot{p}_1/\dot{\rho}_1 = 0$  and

$$\frac{\dot{p}_2}{\dot{\rho}_2} = -\frac{p_2}{\rho_2} = \frac{A_2}{\rho_2^2} = \frac{A_2}{\frac{A_2}{1+\kappa} + \frac{B_2}{a^6}} = (1 + \kappa) \frac{\dot{p}}{\dot{\rho}} . \quad (32)$$

Since for large times we have  $\dot{p}/\dot{\rho} \rightarrow 1$ , the quantity  $\dot{p}_2/\dot{\rho}_2$  may be larger than unity which at the first glance seems to imply a superluminal sound propagation. However,  $\dot{p}_2/\dot{\rho}_2$  is a formal quantity only, which does not describe any

propagation phenomenon. The adiabatic sound speed (31) may also be split with respect to the effective pressures according to (8) with

$$\frac{\dot{p}_1^*}{\dot{\rho}_1} = \frac{\dot{p}}{\dot{\rho}} \quad \text{and} \quad \frac{\dot{p}_2^*}{\dot{\rho}_2} = \frac{\dot{p}}{\dot{\rho}} . \quad (33)$$

These effective sound velocities of the components coincide with the total sound velocity.

## 4 Non-adiabatic pressure perturbations

The Chaplygin in its two-component interpretation of the previous section belongs to a class of models with

$$\dot{\kappa} = 0 \quad \text{and} \quad \rho_1 + p_1^* = \kappa (\rho_2 + p_2^*) . \quad (34)$$

Generally, the components may have (not necessarily constant) equations of state

$$p_1 = w_1 \rho_1 \quad \text{and} \quad p_2 = w_2 \rho_2 . \quad (35)$$

For the total equation of state parameter  $w$  it follows that

$$w = \frac{w_2 + \kappa w_1}{1 + \kappa} \quad \text{and} \quad p = w \rho . \quad (36)$$

For our special Chaplygin gas case these quantities are

$$w_1 = 0, \quad w_2 = -\frac{A_2}{\rho_2^2}, \quad w = -\frac{A}{\rho^2} . \quad (37)$$

Under the condition (34) the difference of the individual sound speeds is

$$\frac{\dot{p}_2}{\dot{\rho}_2} - \frac{\dot{p}_1}{\dot{\rho}_1} = \frac{1 + \kappa}{\kappa} \frac{\dot{\Pi}}{\dot{\rho}_2} . \quad (38)$$

Introducing now matter perturbations in terms of the quantities defined in subsection 2.2 and allowing the density ratio  $\kappa$  to fluctuate, the difference between the fractional energy density perturbations which describes entropy perturbations becomes directly proportional to  $\hat{\kappa}$ :

$$D_2 - D_1 = -\frac{1}{1 + w} \frac{\hat{\kappa}}{\kappa} . \quad (39)$$

This demonstrates, that a perturbation of the density ratio is essential for entropy perturbations to occur. Since the background ratio  $\kappa$  also determines the relation between the equation of state parameters  $A$  and  $A_2$  in (19) a further

specification may be performed. We shall assume from now on the parameter  $A_2$  to be a true constant, i.e.,  $\dot{A}_2 = 0$ . This choice implies

$$\frac{\hat{A}}{A} = \frac{\hat{\kappa}}{1 + \kappa} . \quad (40)$$

Within this model a fluctuation of the density ratio is equivalent to a fluctuation of the equation of state parameter  $A$ . Any fluctuation of  $\kappa$  or  $A$  generates a non-adiabatic pressure fluctuation

$$P - \frac{\dot{p}}{\dot{\rho}} D = \frac{w}{1 + w} \frac{\hat{\kappa}}{1 + \kappa} = \frac{w}{1 + w} \frac{\hat{A}}{A} . \quad (41)$$

We conclude that perturbing the equation of state parameter  $A$  represents a way to introduce non-adiabatic pressure perturbations in a Chaplygin gas. It is expedient to recall that in an underlying string theoretical formalism the parameter  $A$  is connected with the interaction strength of d-branes [5]. Hence, a fluctuating  $A$  corresponds to fluctuating interactions in string theory. An alternative, perhaps more transparent way of understanding the meaning of the fluctuation of the parameter  $A$  is to consider the Born-Infeld action that leads to the Chaplygin gas. The Lagrangian density in this case takes the form [16, 17]

$$L = \sqrt{-g} V(\phi) \sqrt{-\det[g_{\mu\nu} + \phi_{;\mu} \phi_{;\nu}]} , \quad (42)$$

where  $V(\phi)$  is a potential term. This Lagrangian density leads to the Chaplygin gas equation of state for [9, 11]

$$V(\phi) = \sqrt{A} . \quad (43)$$

For  $\phi = \phi_0 + \hat{\phi}$ , the fluctuation in  $A$  is given by

$$\frac{\hat{A}}{A} = 2 \left( \frac{V'}{V} \right)_{\phi=\phi_0} \hat{\phi} , \quad (44)$$

where the prime denotes the derivative with respect to  $\phi$ . Consequently, fluctuations of  $A$  are allowed if the scalar field  $\phi$  is not in the minimum of the potential. It has been shown that there are configurations for which a sufficiently flat potential (equivalent to an almost constant  $A$ ) admits accelerated expansion [17].

Given that  $w < 0$ , any  $\hat{A} > 0$  will reduce the adiabatic pressure perturbations. For definite statements further assumptions about  $\hat{A}$  (or  $\hat{\kappa}$ ) are necessary since otherwise the problem is undetermined. The structure of (41) motivates a choice  $\hat{A}/A = \mu (1 + w) D$ , for which

$$P = c_{eff}^2 D , \quad c_{eff}^2 \equiv \frac{\dot{p}}{\dot{\rho}} (1 - \mu) . \quad (45)$$



Any  $\mu$  in the range  $0 < \mu \leq 1$  leads to an effective sound speed square  $c_{eff}^2$  which is reduced compared to the adiabatic value  $\dot{p}/\dot{\rho} = -w$ . It is interesting to note that the relation

$$\hat{p} = (1 - \mu) \frac{\dot{p}}{\dot{\rho}} \hat{\rho} = - (1 - \mu) \frac{p}{\rho} \hat{\rho} ,$$

which is a different way of writing (45), coincides with that of a generalized Chaplygin gas with an equation of state  $p = -C/\rho^{(1-\mu)}$  with (a true) constant  $C$ . For a generalized Chaplygin gas one would, however, also have  $\dot{p} = - (1 - \mu) \frac{p}{\rho} \dot{\rho}$  and hence adiabatic perturbations only. In a sense, our strategy to consider perturbations  $\hat{A} \neq 0$  while  $\dot{A} = 0$ , implies that the medium behaves as a Chaplygin gas in the background and shares features of a generalized Chaplygin gas on the perturbative level.

At this point also the role of the assumption  $\hat{A}_2 = 0$  that precedes eq. (40) becomes clear. We mentioned at the end of subsection 3.1 that the decomposition of a given Chaplygin gas into another Chaplygin gas interacting with matter may be repeated again and again. In a subsequent step  $A_2$  would play the role that  $A$  played so far. Hence, the assumption  $\hat{A}_2 = 0$  ensures that no further non-adiabatic contributions will appear.

## 5 Conclusions

We have demonstrated that any Chaplygin gas in a homogeneous and isotropic Universe can be regarded as being composed of another Chaplygin gas which is in interaction with a pressureless fluid such that that the background ratio of the energy densities of both components remains constant. This two-component interpretation allowed us to establish a simple model for non-adiabatic pressure perturbations as the result of a specific internal structure of the substratum. Within the one-component picture this internal structure manifests itself as a spatial fluctuation of the parameter  $A$  in the Chaplygin gas equation of state  $p = -A/\rho$ . Such fluctuations, which can be traced back to fluctuations of a tachyon field potential, modify the adiabatic sound speed of the medium which may shed new light on the status of Chaplygin gas models of the cosmic substratum.

### Acknowledgements

We thank the organizers of the *I Brazilian Workshop on Dark Energy/Matter*, held in Joinville, Brazil, for the hospitality and for the nice ambiance during the meeting. This work has been partially supported by CNPq (Brazil).

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